

Maximum Entropy Production and Non-Gaussian Climate Variability

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ABSTRACT

Earth's atmosphere is in a state far from thermodynamic equilibrium. For example, the large scale equator-to-pole temperature gradient is maintained by tropical heating, polar cooling, and a midlatitude meridional eddy heat flux predominantly driven by baroclinically unstable weather systems. Based on basic thermodynamic principles, it can be shown that the meridional heat flux, in combination with the meridional temperature gradient, acts to maximize entropy production of the atmosphere. In fact, maximum entropy production (MEP) has been successfully used to explain the observed mean state of the atmosphere and other components of the climate system. However, one important feature of the large scale atmospheric circulation is its often non-Gaussian variability about the mean. This paper presents theoretical and observational evidence that some processes in the midlatitude atmosphere are significantly non-Gaussian to maximize entropy production. First, after introducing the basic theory, it is shown that the skewness of sea surface winds, damped by nonlinear surface drag, are consistent with the MEP principle. Then it is pointed out that the observed k^{-3} wavenumber spectrum of long planetary waves in the midlatitudes can be roughly explained by maximizing the meridional eddy heat flux (and related entropy production) of unstable baroclinic Eady waves. Finally observational evidence is presented that the meridional eddy heat flux is increased by non-Gaussian variability in meridional wind and temperatures anomalies.

1. Introduction

The atmospheric system is a high-dimensional and highly complex dynamical system with very many nonlinearly interacting modes. One important feature of the large scale (predominantly low-frequency) atmospheric circulation is its non-Gaussianity. Understanding non-Gaussianity has become an important objective in weather/climate research, because weather and climate risk assessment depends on knowing and understanding the tails of probability distributions. In particular, there is broad consensus that the most hazardous effects of climate change are related to a potential increase (in frequency and/or intensity) of extreme weather and climate events populating the tails of often non-Gaussian distributions.

Various mechanisms behind the observed atmospheric non-Gaussian statistics have been proposed. The mechanisms are multifaceted (e.g., nonlinear dynamics, multiplicative noise, cross-frequency coupling) and scattered in the literature. Sura and Hannachi (2015) provide a comprehensive review, focusing on atmospheric synoptic and low-frequency variability. However, while there are many mechanisms contributing to non-Gaussian atmospheric statistics, there does not exist a deeper grand principle beyond the diversity of phenomena. Yet it is not unreasonable to believe that there may exist a deeper grand principle beyond the diversity of phenomena. For example, one possibility is that atmospheric and/or climate statistics are collectively non-Gaussian to maximize a specific functional of the weather/climate system. One possible candidate to explore would be the entropy production (e.g., Paltridge 1975; Ozawa et al. 2001, 2003; Kleidon 2009). That is, the climate system may exhibit a specific collective non-Gaussianity to maximize the overall entropy production.

Maximum entropy production (MEP) has already been successfully (and extensively) used to explain the observed *mean* state of the atmosphere and climate, such as the meridional distribution of mean air temperature, cloud cover, and meridional heat transport (Paltridge 1975). While the initial idea goes back to Paltridge (1975), excellent summaries/reviews have been provided by many authors (e.g., Ozawa et al. 2001, 2003; Kleidon 2009). At this point it is important to realize that, despite many attempts, MEP has (so far) no generally accepted theoretical basis. That is, while it makes physical sense that a system *not* in its unique thermodynamic equilibrium state wants to get there as fast as possible by maximizing entropy production, this physically reasonable statement does not constitute a universal proof that the MEP principle is generally valid in nature. A better term would be MEP conjecture, but for the sake of simplicity we use the words “principle” and “conjecture” interchangeably.

In this paper we propose that MEP may also point a way forward towards a unified perspective of *non-Gaussianity* in weather and climate variability. The basic theory (equations governing the time rate of change of entropy of an open fluid system) is derived and explained in section 2. In contrast to previous work the theory is formulated to highlight the potential impact of probability distributions on entropy production. In section 3 we use MEP to study a simplified conceptual model of the midlatitude atmospheric circulation. The model consists of a midlatitude volume above the boundary layer. The baroclinically unstable atmosphere within that volume, which is frictionally damped by the surface drag due to turbulence in the atmospheric boundary layer, transports heat poleward to keep the average temperatures in the tropics and in the polar zone constant. As a first result we will

see that the observed skewness of sea surface winds, damped by nonlinear surface drag, is consistent with MEP. While it is already known that the product $\overline{F}\Delta T$ of the meridional heat flux \overline{F} and the meridional temperature gradient ΔT has to be maximized to be in accordance with the MEP principle¹, a novel finding is that the observed k^{-3} wavenumber spectrum of long waves can be roughly explained by maximizing the meridional eddy heat flux $\overline{v'T'}$ of unstable baroclinic Eady waves. Next it is shown that the meridional eddy heat flux is increased by non-Gaussian variability in meridional wind and temperatures anomalies v' and T' . Overall that means, as discussed in the final section 4, that MEP is consistent with the observed non-Gaussian variability in the midlatitude atmosphere, and may point a way forward towards a unified perspective of non-Gaussianity in weather and climate variability.

2. Theory

a. Basics

In the following let us derive and discuss the basic equations governing the time rate of change of entropy of an *open* fluid system [in the style of Ozawa et al. (2001)]. Here *open* means that the system exchanges heat and momentum, but not overall mass, with its surrounding. The starting equation is the time derivative

$$\dot{S}_{sys} = \frac{d}{dt} \left[\int \rho s dV \right] = \int \frac{\partial(\rho s)}{\partial t} dV + \int \rho s v dA, \quad (1)$$

¹Note that if the meridional temperature gradient ΔT is assumed to be constant, it is the meridional heat flux \overline{F} alone that has to be maximized.

where ρ is the density of the fluid, s is the entropy per unit mass, v is the normal (positive outward) component of the fluid velocity at the surface, V is the volume of the system, and A is the surface bounding the system. It is important to note that the boundary flux term $\int \rho s v dA$ is needed to maintain the overall (i.e., integrated) mass of the otherwise open fluid system. In the discussion of entropy production by Landau and Lifshitz (1966) only the volume integral appears because they consider a closed system. That is, in Landau and Lifshitz (1966) the mass of the systems is automatically conserved because there is no exchange with its surrounding at all (that is, v in the boundary flux term is zero by construction).

Next, using the continuity equation $\partial\rho/\partial t = -\nabla \cdot (\rho\mathbf{v})$ we obtain

$$\frac{\partial(\rho s)}{\partial t} = \rho \frac{\partial s}{\partial t} + s \frac{\partial \rho}{\partial t} = \rho \frac{\partial s}{\partial t} - \nabla \cdot (\rho s \mathbf{v}) + \rho \mathbf{v} \cdot \nabla s , \quad (2)$$

which we can use, together with Gauss' theorem, to rewrite Eq. (1) to get

$$\dot{S}_{sys} = \int \rho \left[\frac{\partial s}{\partial t} + \mathbf{v} \cdot \nabla s \right] dV = \int \rho \frac{ds}{dt} dV . \quad (3)$$

Note that the expression in square brackets is just ds/dt , the total derivative of the entropy per unit mass following the fluid motion. However, we know that $ds = dq/T$, where dq is the heat flux (per unit mass) into the small volume element dV and T is its temperature. In addition, from the first law of thermodynamics we know that $dq = du + pd(1/\rho)$ (here u is the internal energy per unit mass and p is pressure). Therefore,

$$\frac{ds}{dt} = \frac{1}{T} \left(\frac{du}{dt} + p \frac{d(1/\rho)}{dt} \right) . \quad (4)$$

Using this in Eq. (3), the fact that $du/dt = \partial u/\partial t + \mathbf{v} \cdot \nabla u$, and the continuity equation in

the form $d(1/\rho)/dt = (1/\rho)\nabla \cdot \mathbf{v}$, we obtain

$$\dot{S}_{sys} = \int \frac{1}{T} \left[\rho \frac{\partial u}{\partial t} + \rho \mathbf{v} \cdot \nabla u + p \nabla \cdot \mathbf{v} \right] dV . \quad (5)$$

by using

$$\rho \frac{\partial u}{\partial t} + \rho \mathbf{v} \cdot \nabla u = \frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u \mathbf{v}) \quad (6)$$

and $u = cT$ (where c is the specific heat at constant volume) we finally get

$$\dot{S}_{sys} = \int \frac{1}{T} \left[\frac{\partial(\rho c T)}{\partial t} + \nabla \cdot (\rho c T \mathbf{v}) + p \nabla \cdot \mathbf{v} \right] dV \equiv \int \frac{Q}{T} dV . \quad (7)$$

Equation (7) describes the rate of entropy change of an open fluid system in terms of pressure, temperature, density, and velocity fields. The term in square brackets is the local *adiabatic* heating rate per unit volume Q which can be rewritten in terms of the convergence of a diabatic heat flux density \mathbf{F} due to fluid motion (turbulence) and the heating rate (per unit volume) Φ representing the rate of dissipation of kinetic energy by viscosity:

$$Q = -\nabla \cdot \mathbf{F} + \Phi . \quad (8)$$

The heat flux \mathbf{F} includes all diabatic heat transport processes associated with turbulent fluid motion (i.e., heat conduction, latent heat transport). It is important to note that \mathbf{F} does *not* include the advective heat flux because advection (i.e. coherent motion of fluid) is intrinsically a reversible process (one can reverse the heat transport by reversing the movement). We could include a reversible advective heat flux in \mathbf{F} but it would results in *no* contribution to entropy production of the whole system: moving the internal energy (i.e. heat) of fluid from one place to another alone will not change the system's overall entropy. However, the advection of, for example, hot fluid into a colder region results in entropy

production by heat conduction across the frontal regions with strong temperature gradients (as we will see from the overall entropy production equation next).

While Eq. (7) describes the entropy production of the open fluid system, the entropy of the surrounding system will also increase by the heat flux \mathbf{F} from the fluid system through the boundary. The rate of entropy change of the surrounding system \dot{S}_{surr} is given by the surface integral

$$\dot{S}_{surr} = \int \frac{F}{T} dA , \quad (9)$$

where F is the normal component (defined as positive outward) of the heat flux \mathbf{F} . Now we are in the position to formulate the budget for entropy production (due to turbulence in the fluid system) of the whole system as the sum of Eqs. (7) and (9):

$$\dot{S}_{turb} = \int \frac{-\nabla \cdot \mathbf{F} + \Phi}{T} dV + \int \frac{F}{T} dA . \quad (10)$$

Using Gauss' theorem we can rewrite the surface integral as

$$\int \frac{F}{T} dA = \int \nabla \cdot \left(\frac{\mathbf{F}}{T} \right) dV = \int \frac{\nabla \cdot \mathbf{F}}{T} dV + \int \mathbf{F} \cdot \nabla \left(\frac{1}{T} \right) dV , \quad (11)$$

to obtain our final entropy production equation for the whole system

$$\dot{S}_{turb} = \int \mathbf{F} \cdot \nabla \left(\frac{1}{T} \right) dV + \int \frac{\Phi}{T} dV . \quad (12)$$

Equation (12) describes the rate of entropy change of the whole system due to motion/turbulence in the fluid system. The first term is the rate of entropy increase by heat conduction (i.e. thermal dissipation), and the second is that by viscous dissipation.

Finally, let us look into a steady state system in some more detail. Then the entropy of the fluid system remains constant and Eq. (10) becomes

$$\dot{S}_{turb,st} = \int \frac{F}{T} dA , \quad (13)$$

where the subscript *st* denotes that the fluid systems is in a steady state. This equation tells us that in a steady state the entropy produced by irreversible processes (i.e., thermal and viscous dissipation) in a turbulent fluid is entirely discharged to the surrounding system through a boundary heat flux F . That is, in a steady state we have

$$\dot{S}_{turb,st} = \int \mathbf{F} \cdot \nabla \left(\frac{1}{T} \right) dV + \int \frac{\Phi}{T} dV = \int \frac{F}{T} dA = \text{Maximum} , \quad (14)$$

where we ultimately also *assumed* that the fluid system maximizes the entropy production. It has been already mentioned in the introduction that, while it makes physical sense that a system not in its unique thermodynamic equilibrium state wants to get there as fast as possible by maximizing entropy production, this statement does not constitute a universal proof that the MEP principle is generally valid. However, it is possible to show that the global distribution of key climate variables and several maximum transport properties suggested in the literature are consistent with Eq. (14) (Paltridge 1975; Ozawa et al. 2001, 2003), making MEP a physically reasonable tool for climate research.

b. Maximum entropy production: probability distributions

It should be noted that Eq. (14) describes the instantaneous state of the fluid system. However, describing a turbulent fluid by its instantaneous state does not make much statistical sense. That is, we should interpret Eq. (14) in a time averaged framework:

$$\bar{\dot{S}}_{turb,st} = \int \overline{\mathbf{F} \cdot \nabla \left(\frac{1}{T} \right)} dV + \int \overline{\left(\frac{\Phi}{T} \right)} dV = \int \overline{\left(\frac{F}{T} \right)} dA = \text{Maximum} , \quad (15)$$

where the overbar denotes a time mean. Remember that the time mean \bar{x} of the variable x can also be written in terms of its probability density function (PDF) $p(x)$: $\bar{x} = \int xp(x)dx$.

That means that we can rewrite Eq. (15) in terms of the PDFs of the entropy increases due to thermal dissipation, viscous dissipation, and boundary heat fluxes. If we define the new variables $X_1 \equiv \mathbf{F} \cdot \nabla (1/T)$, $X_2 \equiv \Phi/T$, and $X_3 \equiv F/T$ we can rewrite the MEP proposition as

$$\begin{aligned}\bar{\dot{S}}_{turb,st} &= \int \left(\int X_1 p(X_1) dX_1 \right) dV + \int \left(\int X_2 p(X_2) dX_2 \right) dV \\ &= \int \left(\int X_3 p(X_3) dX_3 \right) dA \\ &= \text{Maximum} .\end{aligned}\tag{16}$$

In this equation it is apparent that the entropy production depends on the PDFs of thermal dissipation X_1 , viscous dissipation X_2 , and boundary heat fluxes X_3 . Let us now apply the MEP principle (16) to explain some of the non-Gaussian features observed in the midlatitude atmosphere.

3. Applications

In the following discussion we consider a simplified conceptual model of the atmospheric circulation depicted in Fig. 1. The atmosphere is composed of three regions: a tropical zone, the midlatitudes, and a polar zone. The average temperature in the tropics is T_{trop} and the average temperature in the the polar zone is T_{pole} . In the tropics there is a net input of radiation/heat, whereas in polar regions there is a net output. To keep the average temperatures in the tropics and in the polar zone constant the midlatitude atmosphere transports heat poleward through its (predominantly horizontal) circulation, characterized

through zonal and meridional velocities u and v , and temperature T . The poleward heat flux consists of a flux from the tropics into the midlatitudes F_{TM} , and a flux from the midlatitudes into the polar zone F_{MP} . Below the free midlatitude atmosphere we have a turbulent boundary layer that exerts a drag on the surface winds and induces a dissipative heating rate Φ .

In the following we first consider the entropy production through viscous dissipation in the boundary layer. Then we study entropy production in a midlatitude volume (the shaded region) above the boundary layer.

a. Viscous dissipation

As a simple example let us consider a well-mixed atmospheric boundary layer where the wind speeds (and temperature) are approximately independent of height. According to Monin-Obukhov similarity theory, the zonal and meridional components of the surface stress vector are $\tau_x = \rho c_D u \sqrt{u^2 + v^2}$ and $\tau_y = \rho c_D v \sqrt{u^2 + v^2}$, where u and v are the zonal and meridional surface wind components, c_D is the (nondimensional) drag coefficient, and ρ is the density of air. Then the rate of velocity changes (now in vector notation) due to viscous dissipation are

$$\frac{\partial \mathbf{u}}{\partial t} = \dots - \frac{c_D}{h} \mathbf{u} |\mathbf{u}| , \quad (17)$$

where h is the depth of the boundary layer (the dots symbolize all the other terms in the momentum budget). As the energy lost to viscosity is ultimately converted to heat, the

(instantaneous) viscous heating rate per unit volume in the boundary layer is

$$\Phi = \frac{\rho_{CD}}{h} \mathbf{u}^2 |\mathbf{u}| . \quad (18)$$

Then the averaged MEP proposition (15, 16) simplifies to

$$\bar{S}_{turb,st} = \int \left(\frac{\Phi}{T} \right) dV \propto \int \overline{\mathbf{u}^2 |\mathbf{u}|} dV = \text{Maximum} . \quad (19)$$

That is, locally surface winds have to maximize $\overline{\mathbf{u}^2 |\mathbf{u}|}$. If, for the sake of conceptual simplicity, we also assume a monodirectional wind u , we obtain

$$\overline{\mathbf{u}^2 |\mathbf{u}|} = \overline{u^2 |u|} = \int u^2 |u| p(u) du = \text{Maximum} . \quad (20)$$

To study this equation we use a general parametric PDF $p(u)$ that has been shown to describe non-Gaussian atmospheric variability very well (Sardeshmukh and Sura 2009; Sura 2011, 2013). The non-Gaussian PDF is generated by a univariate linear stochastic differential equation with correlated additive and linear multiplicative (CAM) noise. In particular, it is the CAM noise that is responsible for skewness and kurtosis of the related PDF. As an example, we use a PDF with a 10 ms^{-1} mean wind and 1 ms^{-1} standard deviation. The non-Gaussianity can be controlled by changing the CAM noise parameter, while keeping mean and standard deviation constant. If the additive and multiplicative noise are positively correlated, the skewness is positive; the reverse (negative skewness) is true for negatively correlated additive and multiplicative noise. If we don't have multiplicative noise, the PDF is Gaussian. Representative plots are shown in Fig. 2. The thick solid line denotes a PDF with negative skewness, the thin solid line is a PDF with positive skewness, and the dashed line represents a Gaussian; all three PDFs have the same mean and variance. Given the PDFs we can calculate the entropy production $\propto \overline{u^2 |u|}$. It turns out that, in general, the PDF

with negative skewness maximizes $\overline{u^2|u|}$. That is, for positive mean winds MEP predicts negative skewness. For negative mean winds MEP favors positive skewness (not shown). Sure enough, this link between mean and skewness is consistent with observations.

For example, Fig. 3 shows the mean, standard deviation, and skewness of NCEP-NCAR zonal sea surface (995 hPa) winds. The mean sea surface zonal wind field displays the familiar tropical easterlies and midlatitude westerlies. The standard deviation shows variability minima in the tropics/subtropics and maxima in the midlatitude storm tracks. The skewness of the zonal wind is generally positive in the tropics and negative in the midlatitudes. In fact, there exists a linear relationship between the mean and skewness of the zonal winds: positive mean winds are related to negative skewness and vice versa. The same relationship holds for meridional winds; see Monahan (2004) for more details.

While here we derived the basic link between mean and skewness of sea surface winds using the MEP proposition, the dynamical principle behind the observed anticorrelation can also be understood rather easily (Monahan 2004). As with MEP, the main building block is the fact that the stress in the boundary layer is nonlinear. Now the basic understanding of the link between the mean and the skewness of the sea surface wind components is straightforward. First suppose that the mean wind is zero and that the fluctuations are equally likely to be positive and negative. Then both the mean and skewness of the resulting wind will be zero. Now suppose that the mean wind is positive. Because of the nonlinear surface drag, positive wind anomalies will experience stronger friction than negative ones. Therefore, the winds will be negatively skewed. The opposite is true if we consider a negative mean wind. That is, the non-Gaussianity of the sea surface winds is due to the nonlinear

boundary layer drag. Over land the situation is a bit more complicated because the boundary layer is influenced more strongly by the diurnal cycle than over sea (He et al. 2010; Monahan et al. 2011).

b. Meridional heat transport

As the next example let us now neglect the effect of the boundary layer and focus on the horizontal heat fluxes (Fig. 1). In our conceptual model of the atmosphere that means that the flux from the tropics into the midlatitudes equals the flux from the midlatitudes into the polar zone: $F_{TM} = F_{MP} \equiv F$. Then the MEP proposition (15, 16) becomes

$$\dot{\bar{S}}_{turb,st} = \frac{\bar{F}}{T_{Pole}} - \frac{\bar{F}}{T_{Trop}} = \frac{\Delta T \bar{F}}{T_{Pole} T_{Trop}} = \text{Maximum} \quad , \quad (21)$$

where $\Delta T = T_{Trop} - T_{Pole}$. That is, the product of the mean meridional heat flux \bar{F} and the meridional temperature gradient ΔT has to be maximized to be in accordance with the MEP principle. If we treat the temperatures in the zones/reservoirs as constant ΔT is also constant, and the mean meridional heat flux \bar{F} alone has to be maximized for the global atmosphere to be in a state of maximum entropy production. However, it is important to note that the tropics to pole temperature difference ΔT is actually a function of the mean heat flux \bar{F} : $\Delta T = \Delta T(\bar{F})$ (Ozawa et al. 2001, 2003). To understand this, consider the extreme case of a motionless atmosphere. In this static state the meridional heat flux vanishes: $\bar{F} \approx 0$. Then the tropics will heat up and the poles will cool down, resulting in an increased (decreased) thermal emission from the tropics (poles), respectively, to close the energy imbalance in each region. Therefore, the temperature difference ΔT will be largest in the static state. If we now allow \bar{F} to increase from zero, ΔT will decrease. On the other

hand, if we have extreme atmospheric mixing resulting in a very large \overline{F} , the temperature difference ΔT will be very small. Overall this means that $\Delta T(\overline{F})$ is a monotonic decreasing function of \overline{F} (Ozawa et al. 2001, 2003). Now, because the entropy production $\overline{S}_{turb,st}$ in (21) is proportional to the product $\Delta T(\overline{F})\overline{F}$, it must have a maximum between the two extreme cases $\overline{F} = 0$ (static state with no mixing) and $\Delta T(\overline{F}) = 0$ (extreme mixing). According to the MEP principle (15, 16), this maximum corresponds to the optimal mean state of the atmospheric circulation. Starting with Paltridge (1975) it has been shown in many papers [see Ozawa et al. (2001, 2003) and Kleidon (2009) for excellent reviews] that the observed mean state of the climate system is indeed maximizing entropy production.

As mentioned earlier, in its original formulation (14, 15) \overline{F} does not include advective heat fluxes because advection is a reversible process. However, in the case of our box model we have to take a closer look. The midlatitude atmosphere is predominantly transporting heat by turbulent advection $\overline{v'T'}$ induced by baroclinic instability. As baroclinic instability is effectively just replacing cold high-density air from high altitudes (and latitudes) with warm low-density air from low altitudes (and latitudes), it is obvious that there is no entropy produced in that process. If the turbulent (eddy) heat flux does not produce entropy, where is the entropy produced in our box model? In a similar model for small scale thermal convection (like in a lava lamp) the MEP equation is identical to (21), where the temperatures are then related to the heating at the bottom and cooling at the top (Ozawa et al. 2001, 2003). Within the bulk of the fluid the convection (i.e., heat advection) does not produce entropy. Only in the thin thermal boundary layers the temperature gradients are, on average, large enough to establish an irreversible heat conduction flux \overline{F} out of the fluid volume that actually

contributes to the entropy production. In our atmospheric model (Fig. 1) we do not have thermal boundary layers between the boxes, so the close analogy breaks down here. However, because we have a mass flux between the boxes due to the atmospheric general circulation (keeping the net masses of our boxes constant, though), we implicitly assumed that tropical and polar boxes are well mixed to have constant temperatures. That is, the box model setup implies that entropy is produced mainly through mixing, and that the entropy production through mixing equals the rate predicted by (21) with advection included in \overline{F} [as in the multiple box model used Paltridge (1975)].

1) MERIDIONAL HEAT TRANSPORT THROUGH BAROCLINIC INSTABILITY

In the midlatitude atmosphere most of the meridional heat transport is done by unstable baroclinic eddies (i.e., weather systems). Therefore, the meridional heat flux is dominated by the eddy covariance of temperature anomalies T' and meridional velocity anomalies v' : $\overline{F} \approx c\overline{v'T'}$, where c is the specific heat at constant volume. While we know that the atmospheric circulation maximizes entropy production and therefore, if we assume for the sake of simplicity that the meridional temperature gradient ΔT is constant, the heat flux \overline{F} , we do not know much about the dynamics behind the maximization. To do that, we need to have a dynamical model representing baroclinic instability in our a midlatitude volume (the shaded region above the boundary layer) in Fig. 1.

The simplest model of baroclinic instability, which represents the essential instability process in its purest and mathematically feasible form, was introduced by Eady (1949). The details of Eady's model can be found in all standard textbooks on dynamical meteorology

and geophysical fluid dynamics (e.g., Holton 2004; Pedlosky 1987; Gill 1982; Vallis 2006). That is, in the following discussion we will use Eady’s model to represent unstable baroclinic waves and the related poleward heat flux in our midlatitude volume. In the previous section we have seen that the tropics to pole temperature gradient is actually a function of the meridional heat flux: $\Delta T = \Delta T(\overline{F})$. However, in the following discussion of the heat flux induced by unstable Eady waves we consider the temperature gradient as constant. That means that we are strictly speaking not maximizing entropy production but the heat flux. In this simplified setting maximizing entropy production and maximizing heat flux are equivalent. Nevertheless, in the real atmosphere these two principles are *not* identical any more because $\Delta T = \Delta T(\overline{F})$. Thus in the following treatment of the Eady model we consider a maximum flux principle that we derived as a special (i.e., simplified) case of the MEP principle, keeping in mind that they are not equivalent.

In a nutshell, the Eady model is a quasi-geostrophic and continuously stratified baroclinic atmosphere in a f -plane channel of width L_y . The basic state density is constant, the vertical shear of the basic state zonal flow is constant ($\partial \bar{u} / \partial z = \Lambda = \text{constant}$), and it has rigid lids at the bottom ($z = 0$) and the top ($z = H$) of the atmosphere. Then the linearized potential vorticity equation and the first law of thermodynamics in terms of the streamfunction $\psi = \bar{\psi} + \psi'$ [i.e., $\partial \psi / \partial y = -u$ (u is zonal wind), $\partial \psi / \partial x = v$ (v is meridional wind), $\partial \psi / \partial z = RT / f_0 H$ (T is temperature), and $R = 287 \text{ J K}^{-1} \text{ kg}^{-1}$ is the gas constant for dry air] are

$$\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) \left(\nabla^2 \psi' + \epsilon \frac{\partial^2 \psi'}{\partial z^2} \right) = 0 \quad (22)$$

and

$$\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) \frac{\partial \psi'}{\partial z} - \frac{\partial \psi'}{\partial x} \frac{\partial \bar{u}}{\partial z} = 0 , \quad (23)$$

where $\epsilon = f_0^2/N^2$ (f_0 is the Coriolis parameter and N is the buoyancy frequency). Solutions can be found in the form $\psi'(x, y, z, t) = \Psi(z) \cos(ly) \exp[ik(x - ct)]$, where $\Psi(z)$ is a complex amplitude and $c = c_r + ic_i$ is a complex phase speed. Obviously, the flow is unstable with a growth rate kc_i if c has an imaginary part $c_i > 0$. The stability criterion can be obtained from the dispersion relation $c = \Lambda H/2 \pm \Lambda H/2 [1 - 4 \coth(\alpha H)/(\alpha H) + 4/(\alpha H)^2]^{1/2}$, where $\alpha^2 = (k^2 + l^2)/\epsilon$: The flow is unstable for $\alpha < \alpha_c$, where the critical value α_c is given by $\alpha_c H/2 = \coth(\alpha_c H/2)$ or $\alpha_c \approx 2.4/H$. For typical atmospheric values ($H = 7$ km, $N = 1.2 \times 10^{-2} \text{ s}^{-1}$, $f_0 = 10^{-4} \text{ s}^{-1}$, $\Lambda = 8 \times 10^{-3} \text{ s}^{-1}$) we obtain $\alpha_c = 0.343 \text{ km}^{-1}$. If we now calculate the zonal wavenumber k of maximum growth rate for a channel width $L_y = 1500$ km and allow only half a wavelength in the meridional direction (i.e., $l = \pi/L_y$), we obtain $\lambda_g \approx 4500$ km (or $k_g = 2\pi/\lambda_g \approx 0.0014 \text{ km}^{-1}$), where λ_g is the wavelength where $c_i k$ has a maximum. Of course, the Eady model has been seminal because this wavelength is similar to the wavelength of observed weather systems in the midlatitudes (see Holton 2004; Pedlosky 1987; Gill 1982; Vallis 2006, for details). The growth rate as a function of k is shown in Fig. 4 (solid line).

We can also calculate the poleward eddy heat flux of unstable Eady waves (averaged over one wavelength λ):

$$\overline{v'T'} = \frac{1}{\lambda} \int_0^\lambda v'T' dx = \frac{c_i \alpha^2 k f_0 H A^2}{2\Lambda R} \cos^2(ly) \exp^2(kc_i t) , \quad (24)$$

where A is an (so far) arbitrary amplitude coefficient of the stream function perturbation. That is, $\overline{v'T'} \propto c_i \alpha^2 k$ from which we can calculate the wavelength of maximum heat transport

$\lambda_h \approx 4200$ km (or $k_h = 2\pi/\lambda_h \approx 0.0015$ km⁻¹), where λ_h is the wavelength where $c_i \alpha^2 k$ has a maximum. The (dimensionless) heat transport as a function of k is shown in Fig. 4 (dotted line); it is arbitrarily scaled to have the same maximum value as the growth rate to make the shape of the graphs easily comparable.

While the maxima $\lambda_g \approx 4500$ km and $\lambda_h \approx 4200$ km are not identical, they are very similar (i.e., have the same order of magnitude). Thus, the maximum heat flux principle (derived from MEP in our simplified setting) is consistent with the observed wavelength of unstable baroclinic eddies in the midlatitude atmosphere. In particular, the wavelength of the fastest growing wave explains more than 95% of the maximum heat transport. Different channel widths are explored in Fig. 5. In Fig. 5a the channel width is decreased to $L_y = 1250$ km, resulting in almost identical wavelengths of maximum entropy production (i.e., eddy heat transport) and maximum growth rate. On the other hand, if the channel width is increased to $L_y = 2000$ km (Fig. 5b), the maxima drift slightly further apart. However, even in that case the wavelength of the fastest growing wave explains more than 90% of the maximum heat transport.

Equation (24) describes the heat flux of a single wave. However, fully developed baroclinic turbulence does consist of a full spectrum of wavenumbers. Therefore, let us assume a turbulent midlatitude atmosphere in which a full spectrum of baroclinically growing eddies is already in place. We also assume that those eddies are circular ($k = l$) and that each of them transports heat poleward according to Eq. (24). To obtain the total heat flux \overline{F} of a full spectrum of waves Eq. (24) has to be summed/integrated over all wavenumbers k :

$$\overline{F} \propto \int_0^\infty A(k)^2 c_i k^3 dk = \text{Maximum} , \quad (25)$$

where we skipped all the constants (including functions of t and y), used $\alpha^2 = 2k^2/\epsilon$ for circular eddies, and already assumed that the total heat flux has to be maximized according to the simplified version of the MEP principle. Note that now A is not a constant but a function of the wavenumber k : $A = A(k)$. Thus, $A(k)^2$ is the spectral density in the wavenumber spectrum of the stream function perturbation (and the related velocity and temperature anomalies). That means Eq. (25) asks us to choose the wavenumber spectrum that maximizes the integral of all wavenumbers. The maximum requires

$$\Delta \int_0^\infty A(k)^2 c_i k^3 dk = \int_0^\infty \Delta [A(k)^2 c_i k^3] dk = 0 , \quad (26)$$

or $A(k)^2 c_i k^3 = \text{const.}$ Therefore, the wavenumber spectrum which maximizes the heat flux is

$$A(k)^2 \propto \frac{1}{c_i k^3} . \quad (27)$$

As we have c_i from the dispersion relation, we can immediately evaluate (27) (Fig. 6). It can be seen that for long waves (small wavenumbers) the spectrum approximately follows a k^{-3} law. For wavelengths shorter than about 4000 km the slope of the spectrum changes towards $k^{-5/3}$. Close to the instability limit (at $k \approx 0.002$ radians/km or a wavelength of about 3000 km) the spectrum bends upward. While we do not expect an overly realistic spectrum from a model as simple as the Eady model, the -3 slope for long waves is astonishingly accurate; an observed atmospheric wavenumber spectrum is shown in Fig. 7 [adapted from Nastrom and Gage (1985)]. There it can be seen that spectra of temperature and velocity have slopes close to $-5/3$ for scales up to about 400 km. At larger scales the spectra steepen to an approximate slope of -3 . That is, we conclude that the observed k^{-3} spectrum for long waves is consistent with heat flux maximization due to baroclinic instability in the

midlatitude atmosphere.

We are, of course, aware of the fact that the observed spectra in Fig. 7 are traditionally explained through the full nonlinear interactions in fully developed quasigeostrophic 2D and 3D turbulence, and that there exists an extensive pool of literature (not referenced here) dealing with the spectra and the related energy and enstrophy cascades. While there is no doubt that full nonlinear models are the appropriate tools to study turbulence it is, however, interesting that a simple linear model of baroclinic instability is able to produce a -3 slope for long waves if we maximize the meridional heat transport. A more detailed study of this phenomenon is part of our ongoing research.

2) MERIDIONAL HEAT TRANSPORT AND NON-GAUSSIAN VARIABILITY

So far we have seen that heat flux maximization, derived from the MEP principle, is capable of explaining some general features of observed wavenumber scales/spectra in the midlatitude atmosphere. Let us now explore if heat flux maximization is also consistent with the observed non-Gaussian variability in the midlatitude free atmosphere. That is, we ask if the midlatitude eddy heat flux $\overline{F} \propto \overline{v'T'}$ is indeed maximized by non-Gaussian variability in v' and/or T' .

In general, the statistics of almost all atmospheric variables are non-Gaussian. However, the origin of non-Gaussianity is not unique and is a topic of active research [Sura and Hannachi (2015) provide a comprehensive review, focusing on atmospheric synoptic and low-frequency variability]. Often skewness S and/or excess² kurtosis K are used to quantify

²Here we use excess kurtosis, defined with respect to that of a Gaussian distribution.

non-Gaussianity: $S(x') \equiv x'^3/\sigma^3$ and $K(x') \equiv x'^4/\sigma^4 - 3$, for anomalies x' with zero mean and standard deviation σ . For example, Perron and Sura (2013) provide a 62-year (1948-2009) climatology of global skewness and (excess) kurtosis at every gridpoint of key atmospheric variables (including meridional wind and temperature anomalies v' and T' , respectively) using daily data from the NCEP-NCAR Reanalysis project (Kalnay and coauthors 1996). The zonally averaged skewness and kurtosis fields of meridional wind and temperature for winter (DJF) and summer (JJA) are shown in Fig. 8 (however, in the following we focus exclusively on the northern hemisphere winter).

For our purpose, let us emphasis several points. The skewness of meridional winds is close to zero in the entire midlatitude northern hemisphere (in winter and summer). The kurtosis of meridional winds is on average slightly positive in the midlatitude northern hemisphere winter. The zonally averaged skewness and kurtosis fields of temperature are a bit more complicated with respect to height and latitude, but we can make the following general statements. Averaged over the entire midlatitude atmosphere the temperature skewness is approximately zero. In addition, the temperature kurtosis is predominantly (on average) negative in the midlatitude northern hemisphere winter. Those general statements can also be visualized by looking at the joint PDF of all midlatitude meridional wind and temperature anomalies v' and T' . To do that we normalize the meridional wind and temperature time series at all midlatitude grid points (approximately $25^\circ\text{N} - 65^\circ\text{N}$ and $900\text{ hPa} - 100\text{ hPa}$) to zero mean and unit standard deviation. We then calculate the joint PDF $p(v', T')$ and the related bivariate Gaussian distribution. The non-Gaussian structure of $p(v', T')$ can now be highlighted by plotting the joint PDF anomaly [i.e., $p(v', T')$ minus the bivariate Gaussian];

the result is shown in Fig. 9a. Of course, the first-order fact to notice is that $p(v', T')$ has a pronounced non-Gaussian structure. There is a pronounced stronger than Gaussian peak around about $v' \approx 0$ and $T' \approx -1$. A secondary peak exists near $v' \approx 0.3$ and $T' \approx +1$. Both peaks are connected by a stronger than Gaussian ridge. On both side of that ridge are weaker than Gaussian regions at about $v' \approx \pm 1$ and $T' \approx 0$. This overall structure results in a marginal distribution $p(v')$ (not shown) with vanishing skewness and positive kurtosis, and a marginal distribution $p(T')$ (not shown) with negligible skewness and negative kurtosis, consistent with the above discussion of Fig. 8 and the analysis of Messori and Czaja (2012).

Let us now analyze if the heat flux $\overline{F} \propto \overline{v'T'}$ is maximized by non-Gaussian variability in v' and/or T' . To do that we calculated the eddy heat flux $\overline{v'T'}$ and the skewness and kurtosis of v' and T' at every midlatitude gridpoint. We then looked at the joint PDF of the individual moments [i.e., $S(v')$, $S(T')$, $K(v')$, $K(T')$] and $\overline{v'T'}$ to find a potential correlation between non-Gaussianity and the eddy heat flux. It turns out that there is no significant correlation between the skewness $K(v')$ and $K(T')$, and $\overline{v'T'}$, which makes sense because $K(v')$ and $K(T')$ are small (as discussed above). However, the eddy heat flux $\overline{v'T'}$ is correlated with the kurtosis $K(v')$ and $K(T')$: The correlation is *positive* between $\overline{v'T'}$ and $K(v')$ (see joint PDF in Fig. 9b), and *negative* between $\overline{v'T'}$ and $K(T')$ (see joint PDF in Fig. 9c). To summarize (and visualize) this behavior we constructed a non-Gaussianity index NGI as $NGI \equiv K(v') - K(T')$ (this index captures the observed correlation between the eddy heat flux and the non-Gaussianity of v' and K') and calculated the joint PDF of this index and the eddy heat flux. The result $p(NGI, \overline{v'T'})$ is shown in Fig. 9d, where it can be seen that there exists a (close to linear) positive correlation between the non-Gaussianity

index and the eddy heat flux. That is, a positive $K(v')$ and a negative $K(T')$ are linked to a high probability of a large positive (i.e., poleward) eddy heat flux $\overline{v'T'}$. On the other hand, a close to Gaussian flow field likely results in a relatively small eddy heat flux.

Overall, the midlatitude eddy heat flux $\overline{F} \propto \overline{v'T'}$ is increased by non-Gaussian variability in v' and T' . In other words, the MEP derived maximum heat flux principle is consistent with the observed non-Gaussian variability and the related meridional eddy heat flux in the midlatitude free atmosphere. While, at this point, we can only speculate (done in the the following conclusions) about specific dynamical mechanisms linking MEP and non-Gaussian variability, MEP (and related flux maximization principles) seems a useful and promising tool to study and potentially better understand non-Gaussian weather and climate variability.

4. Conclusions

In this paper we have presented theoretical and observational evidence that some processes in the midlatitude atmosphere are significantly non-Gaussian to maximize entropy production or related heat fluxes. First we have shown that the skewness of sea surface winds, damped by nonlinear surface drag, are consistent with maximum entropy production (MEP). Then we considered a simplified conceptual model of the atmosphere composed of three regions: a tropical zone, the midlatitudes, and a polar zone (Fig. 1). One novel finding presented in this paper is that the observed k^{-3} wavenumber spectrum of long waves in the midlatitudes can be roughly explained by maximizing the meridional eddy heat flux $\overline{v'T'}$ of

unstable baroclinic Eady waves. Finally we have shown that the meridional eddy heat flux is increased by non-Gaussian variability in meridional wind and temperatures anomalies v' and T' . Overall that means that MEP is consistent with the observed non-Gaussian variability in the midlatitude atmosphere. That is, MEP may point a way forward towards a unified perspective of non-Gaussianity in weather and climate variability.

Why are our findings important and useful? One question often asked with respect to climate change is the following: How does the probability of extreme events changes in a warming climate? More often than not an answer is provided by extensive numerical simulations. Numerical modeling allows us to estimate the statistics of extreme events (the tails of the PDF) by integrating a general circulation model (GCM) for a very long period (e.g., Easterling et al. 2000; Kharin and Zwiers 2005; Kharin et al. 2007). It is obvious that the efforts by the Intergovernmental Panel on Climate Change (IPCC) to understand and forecast the statistics of extreme weather and climate events in a changing climate fall into this category. However, while the numerical modeling is very useful and practical, a weakness lies in the largely unknown ability of a GCM to reproduce the correct statistics of extreme events. Currently, many climate models are calibrated to reproduce the observed first and second moments (mean and variance) of the general circulation of the ocean and atmosphere. Very little is known about the credibility of GCMs to reproduce non-Gaussian statistics.

The MEP proposition allows us to provide a different, more conceptually pleasing, kind of answer, though. The probabilities (i.e., PDFs) of variables and processes under consideration are altered by climate change to maximize entropy production. That is, by changing the

atmosphere's energy balance and related heat fluxes, the climate system adjusts its PDFs to a new MEP state. While, at first sight, this answer is not of much practical value compared to straightforward GCM runs, the MEP principles potentially gives us a tool to gauge the fidelity of global warming simulations in a thermodynamically sound and well established framework. The exploration of this route is part of our ongoing and future research.

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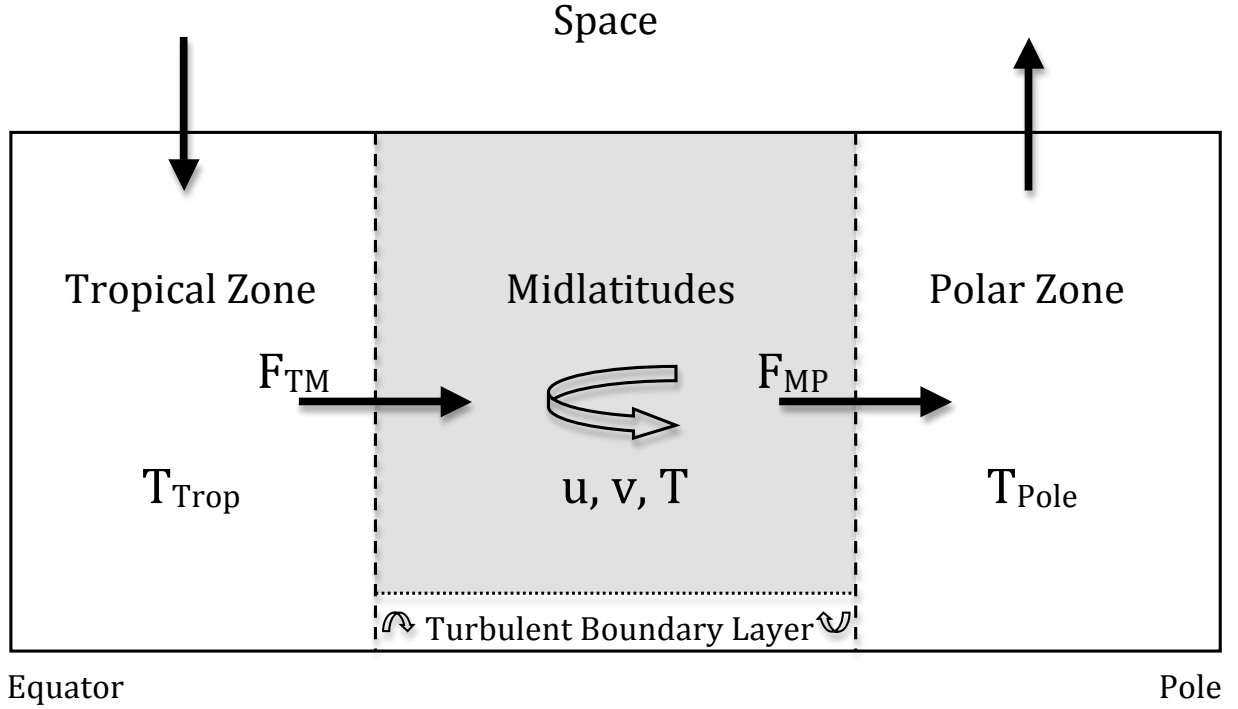


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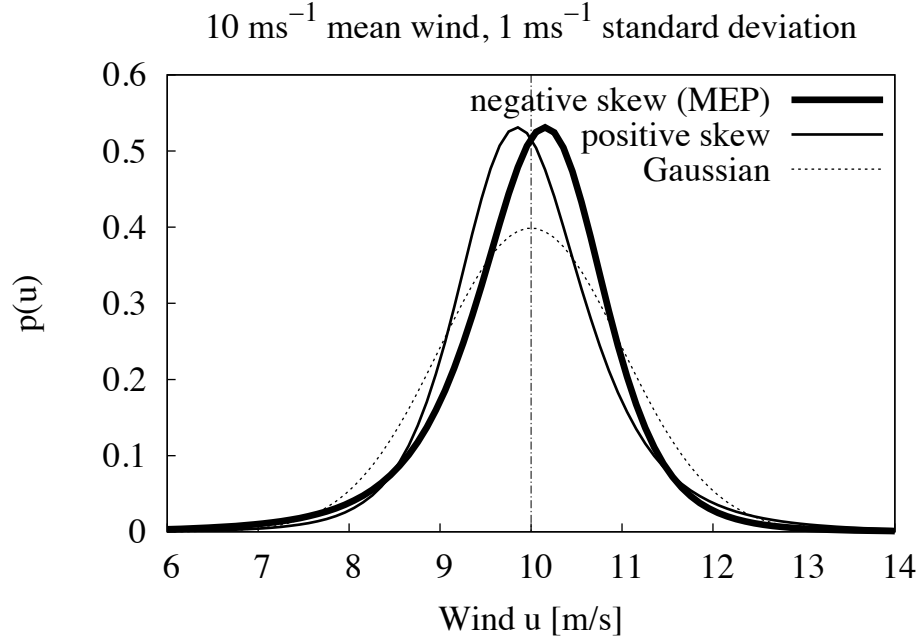


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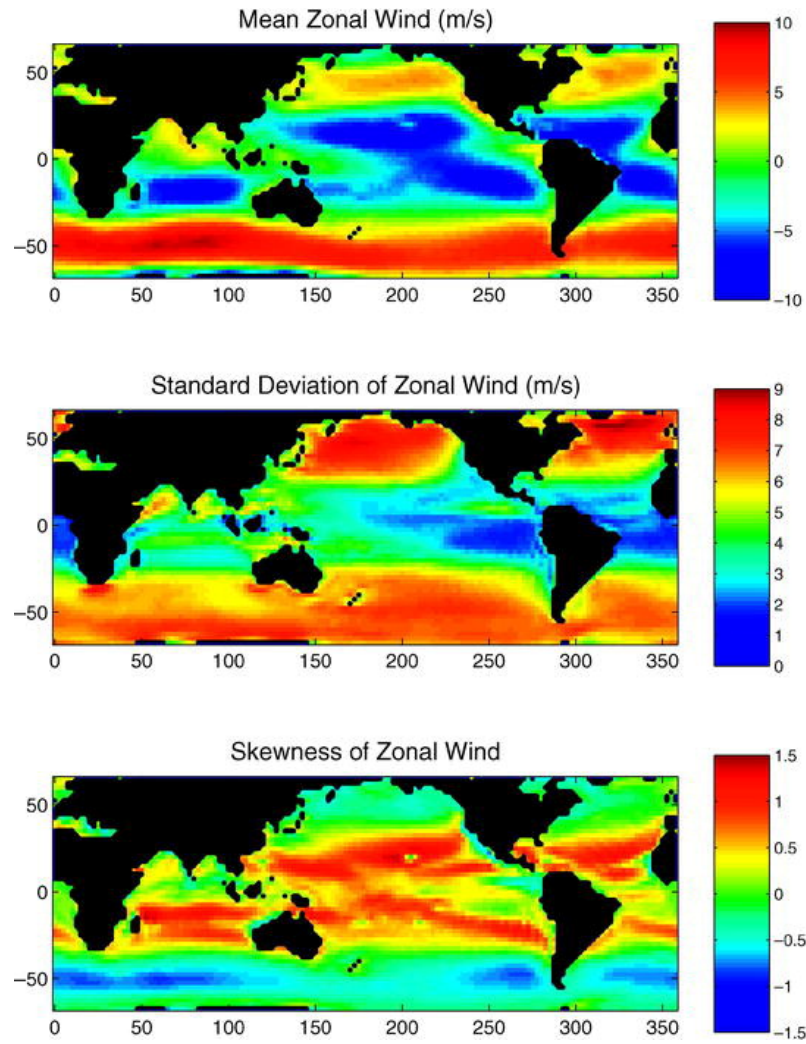


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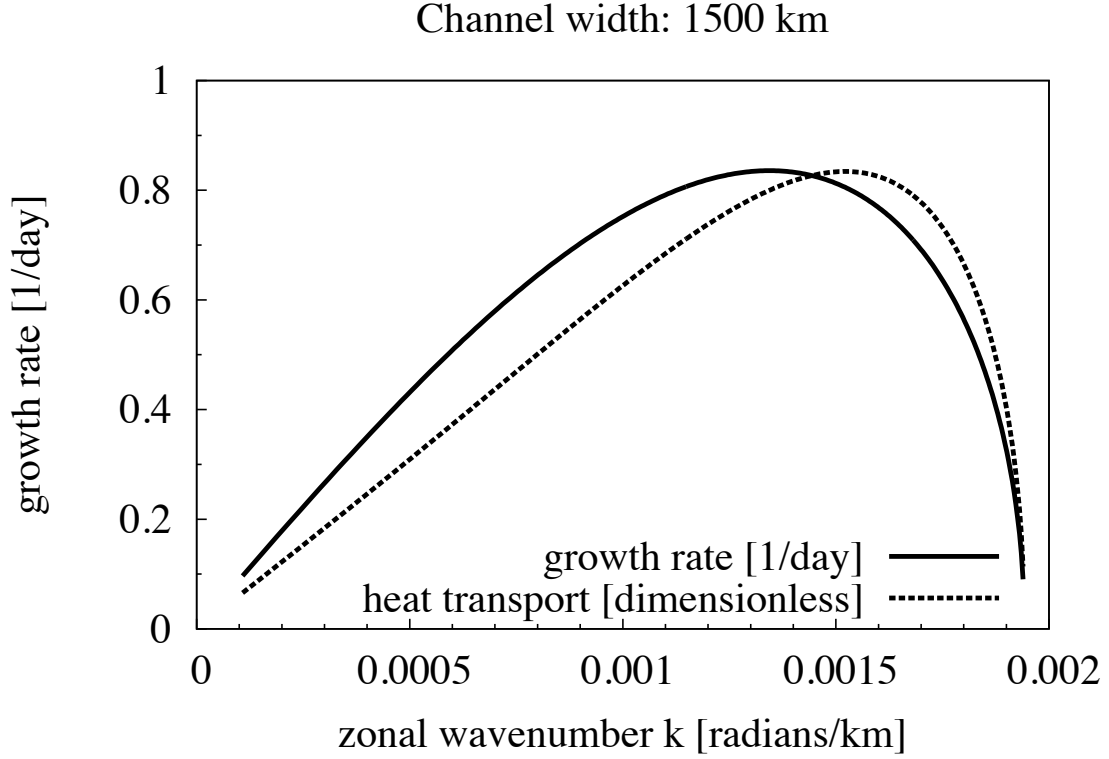
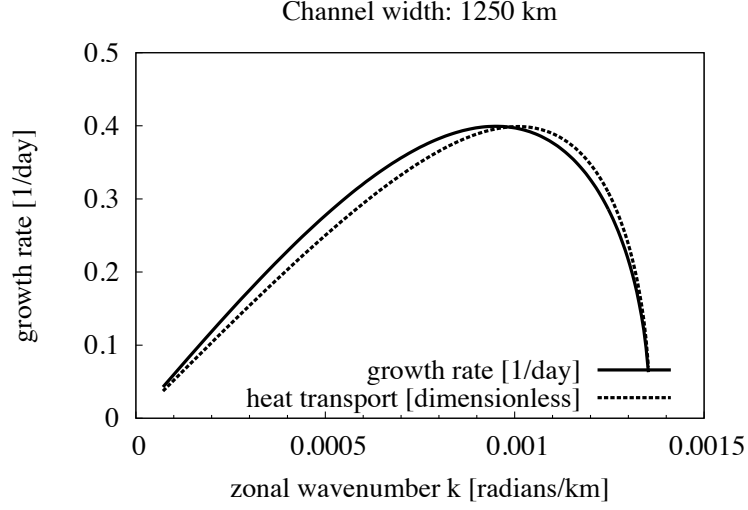


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a)



b)

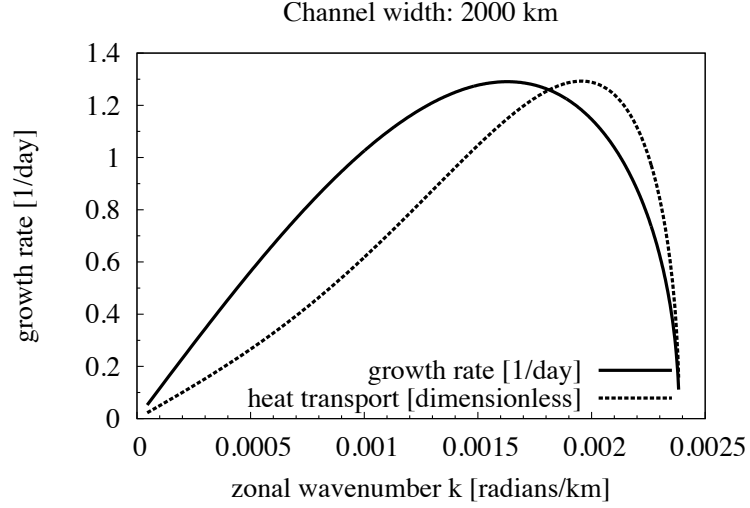


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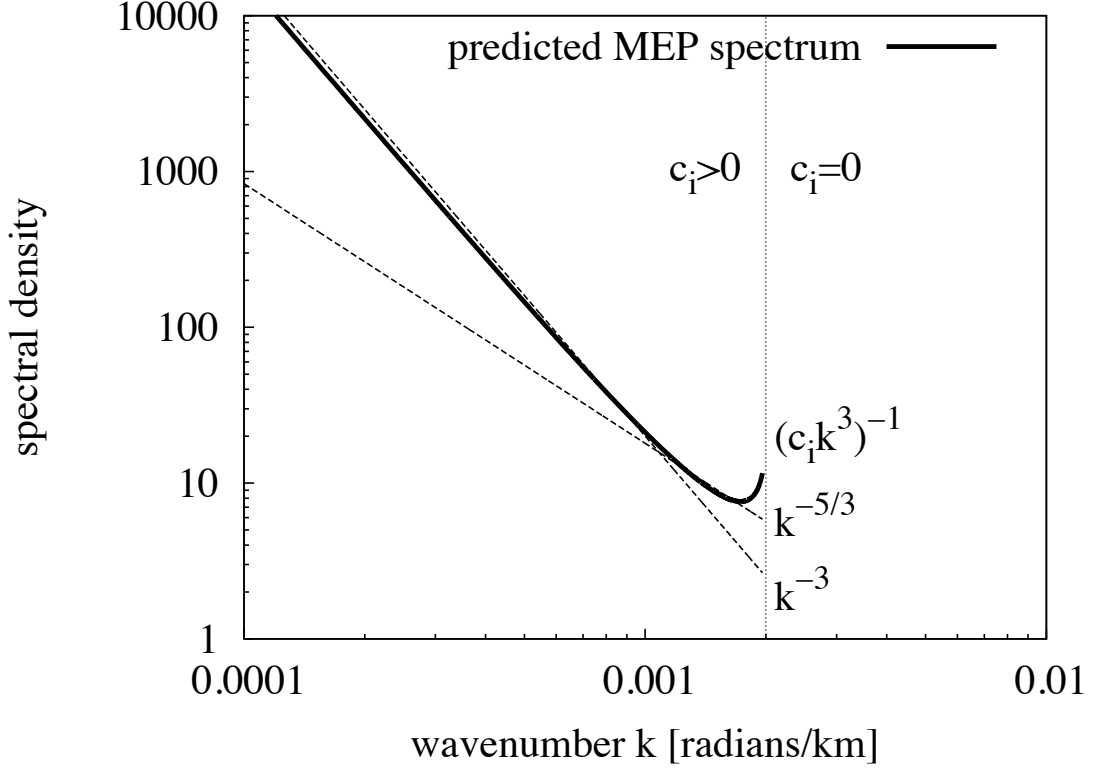


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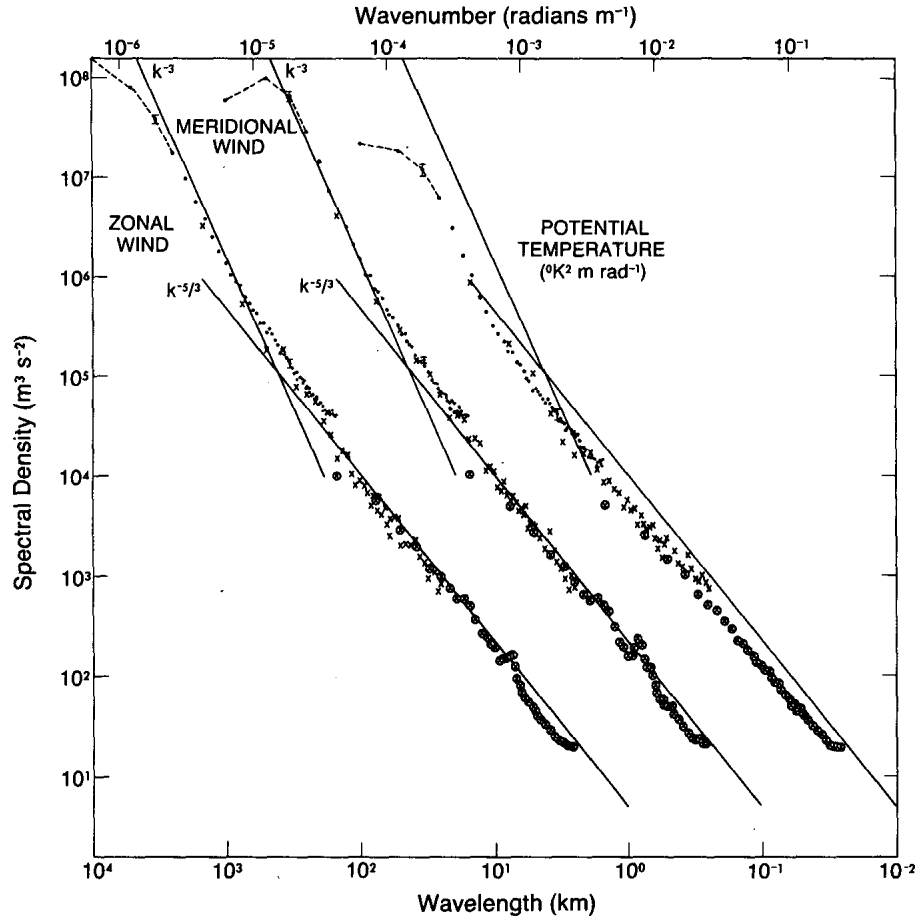


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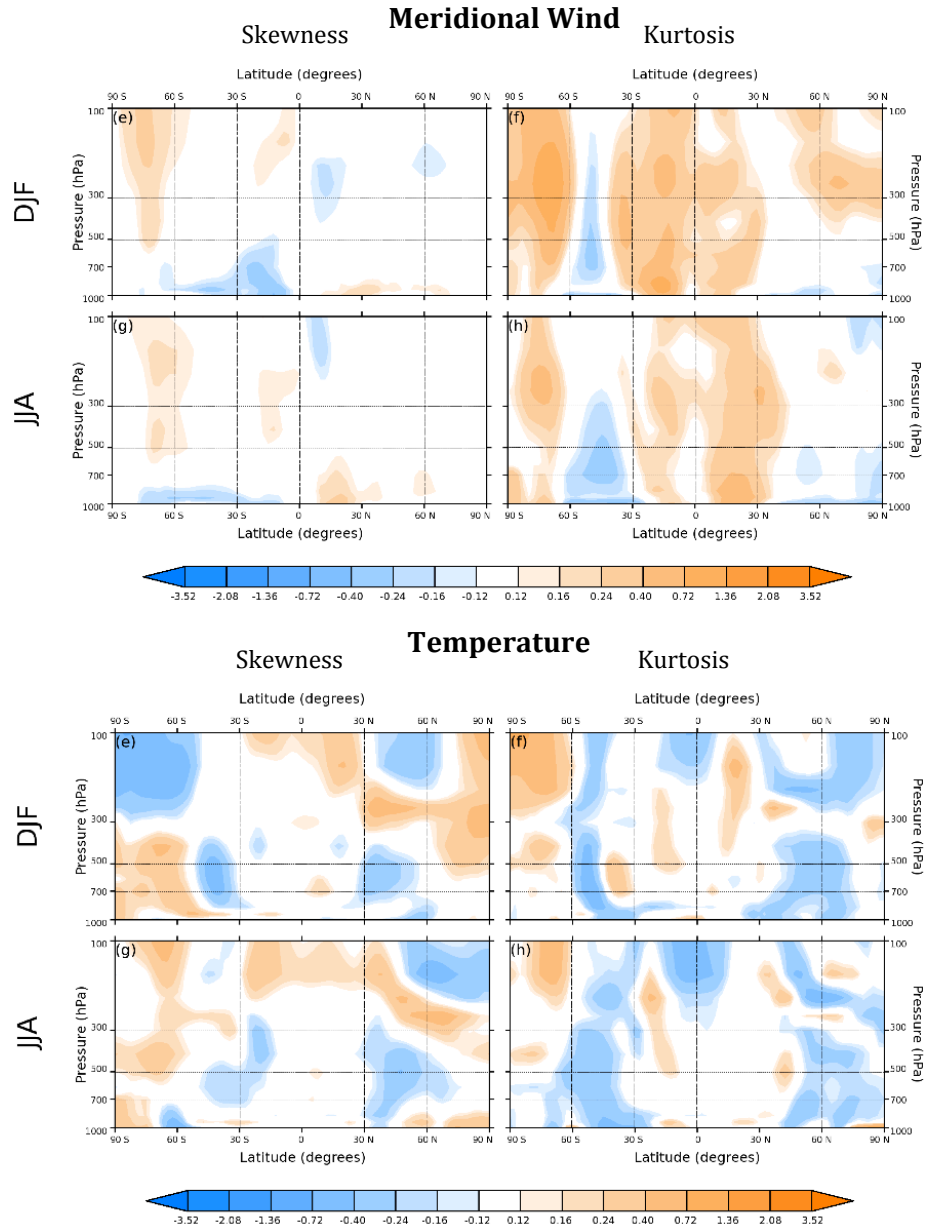


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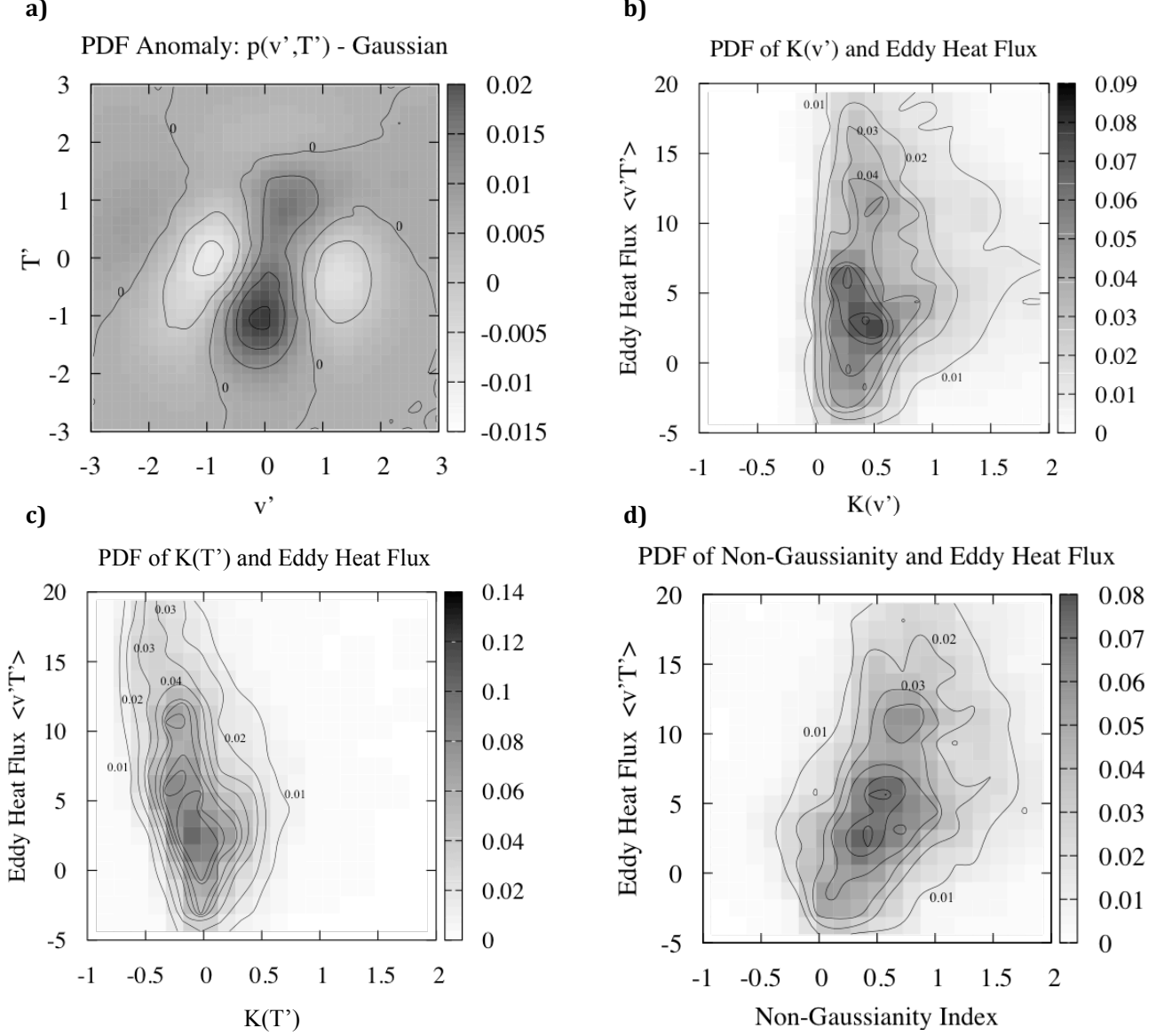


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